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MATHEMATICAL MODELING OF THE HEAT TREATMENT AND COMBUSTION OF A COAL PARTICLE. I. HEATING STAGE

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Mathematical modeling of the heat treatment and subsequent combustion of a coal particle as a multistage process has been carried out. The basic parameters of the following sequential stages of this process have been calculated by approximate-analytic dependences: heating of particles; their drying; yield of volatiles, their ignition and combustion; and burning out of the coke residues. A detailed parametric analysis of the influence of the physical and regime characteristics of the process on the burning mechanism of a coal particle (with the example of coal from the Shivee-Ovoo deposit in Mongolia) has been performed. The conditions for effective burning of a single coal particle as the main element of the whole process in the furnace have been determined.

Keywords: *coal, combustion, heating, drying, burning, coke residue, mathematical model.*

Introduction. Most of the thermal and electric energy in the world is generated with the use of coal. Further development of the power industry is planned also with the use of low-grade coals, including new deposits. Among such deposits is the insufficiently exploited Shivee-Ovoo stripping in Mongolia, for which it is required to conduct a complex of investigations on the heat treatment and burning of coal with the aim of its wide and effective use as an energy source. The basis for calculating the flare combustion of coals is formed by the dependences defining the whole process of combustion of individual coal particles. These processes for particles of natural coals include complex transformations of the organic and mineral parts of the coal matrix, heating, drying, emission and inflammation of volatiles, and burning out of the coke residue. Such detailing requires a complex physical and mathematical modeling.

The most purposeful experiments on burning coal particles were carried out by L. N. Khitrin [1], E. S. Golovina [2], S. Bukhman [3], K. Essenhigh [4], M. Shibaoka [5], and especially by the scientific school of V. I. Babii from VTI [6].

The broad spectrum of factors in the process of heat treatment and combustion of coal particles should be taken into account in the numerical calculations of the whole of the process in the furnace. At the present time, the most generally used programs for this purpose are Fluent [7], Ansys [8], Fire 3D [9], σ-Flow [10], and others.

However, the above factors are most often determined by the method of mathematical modeling of the heat treatment and combustion of single coal particles. This method permits obtaining final approximate-analytical calculation formulas that can be used for detailed parametric analysis of the process and proximate calculations of its parameters. Such as approach has been realized in the present work.

Because of the large body of material used in the process under consideration, the authors have to subdivide it into separate parts corresponding to the sequential and interrelated stages of heat treatment and combustion of a particle connected by a common title. The first part presents the investigations on the radiation-convection heating of the particle. The other stages will be considered in subsequent publications in "Inzhenerno-Fizicheskii Zhurnal."

Heating Stage of the Coal Particle. Under the action of a high-temperature gaseous medium (Fig. 1), the coal particle is heated until on its surface the phase transition (moisture-vapor) temperature is attained.

The reliability of the calculation information markedly increases if the mathematical model takes into account such complicating factors as heat emission, drying of the coal, ignition of volatile substances, their burning out, coke

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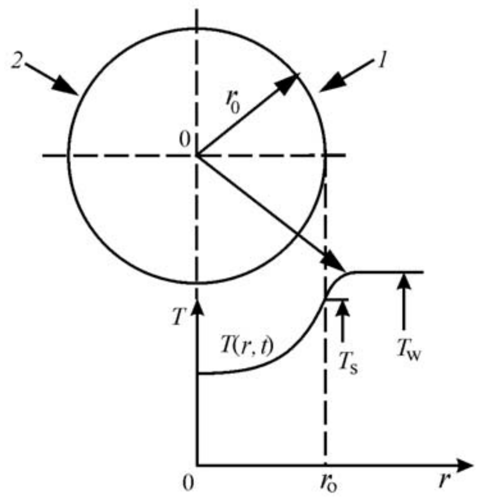

Fig. 1, Scheme of the coal-particle heating under the conditions of radiative-convective heat exchange: 1) radiation; 2) convection.

burning, etc. They introduce nonlinearity into the formulation of the problem, which, naturally, impedes the obtaining of final formulas.

Let us consider a mathematical model of coal-particle heating where nonlinearity is introduced by the boundary condition (radiation-convection heating). The boundary-value problem on determining the nonstationary temperature field is written in the form of the following system of equations:

$$\frac{\partial T(r, t)}{\partial t} = ar^{-2} \frac{\partial}{\partial r}\left[r^2 \frac{\partial T(r, t)}{\partial r}\right], \quad 0 < r < r_0, \quad 0 < t < t_e, \tag{1}$$

with the initial condition

$$T(r, 0) = T_0 \tag{2}$$

and the boundary conditions

$$\lambda \frac{\partial T(r_0, t)}{\partial r} = \sigma [T_m^4 - T^4(r_0, t)] + \alpha [T_m - T(r_0, t)], \tag{3}$$

$$\frac{\partial T(0, t)}{\partial r} = 0. \tag{4}$$

Because of the necessity of performing a generalized analysis, let us transform system (1)–(4) to dimensionless form. As a result, we obtain

$$\frac{\partial \Theta(R, \mathrm{Fo})}{\partial \mathrm{Fo}} = R^{-2} \frac{\partial}{\partial R}\left[R^2 \frac{\partial \Theta(R, \mathrm{Fo})}{\partial R}\right], \quad 0 < R < 1, \quad 0 < \mathrm{Fo} < \mathrm{Fo}_e, \tag{5}$$

$$\Theta(R, 0) = \Theta_0, \tag{6}$$

$$\frac{\partial \Theta(1, \mathrm{Fo})}{\partial R} = \mathrm{Sk}\,[1 - \Theta^4(1, \mathrm{Fo})] + \mathrm{Bi}\,[1 - \Theta(1, \mathrm{Fo})] \equiv \mathrm{Ki}(\mathrm{Fo}), \tag{7}$$

$$\frac{\partial \Theta\,(0, \mathrm{Fo})}{\partial R} = 0\,. \tag{8}$$

Here

$$R = \frac{r}{r_0}\,, \quad \mathrm{Fo} = \frac{at}{r_0^2}\,, \quad \Theta_0 = \frac{T_0}{T_\mathrm{m}}\,, \quad \Theta\,(R, \mathrm{Fo}) = \frac{T\,(r, t)}{T_\mathrm{m}}\,; \quad \mathrm{Sk} = \frac{\sigma r_0 T_\mathrm{m}^3}{\lambda}\,; \quad \mathrm{Bi} = \frac{\alpha r_0}{\lambda}\,.$$

Problem (5)–(8) has no exact analytical solution. The traditional methods for overcoming the difficulties arising in solving it rely on artificial replacement of the nonlinear boundary conditions (7) by a linear condition. The error of the thus-obtained solution is not estimated, as a rule.

Solution of the Boundary-Value Problem (5)–(8) by Reducing It to a Nonlinear Integral Equation. The system of equations (5)–(8) can, following Academician A. N. Tikhonov [11], be reduced to a functional equation and then to a nonlinear integral equation for the surface temperature. With the aid of such an approach it is possible to obtain a formal solution of the boundary-value problem. Although the solution of nonlinear integral equations is rather complicated, reducing the boundary-value problem to them has certain advantages. The latter provides the possibility of establishing immediately the dependence of the problem solution on the domain of its definition, the boundary conditions, and the coefficients, as well as revealing the relations between individual partial solutions.

Such an approach can be realized with the help of the integral Laplace transform. Then the formal solution in the transforms will take the form

$$\Theta_\mathrm{L}\,(R, s) - \Theta_0 = \mathrm{Ki}_\mathrm{L}\,(s)\,F_\mathrm{L}\,(R, s)\,, \tag{9}$$

where

$$F_\mathrm{L}\,(R, s) = \sinh\sqrt{s}\,R\left[\sqrt{s}\,R\left(\cosh\sqrt{s} + s^{-1/2}\sinh\sqrt{s}\right)\right]^{-1}. \tag{10}$$

The inverse transform of (9) in the general case, because of the presence of an infinite number of denominator roots of transform (10), is given as a sum of infinite series, which is extremely difficult in conducting engineering calculations. Under these conditions it is easier to solve the equations for the temperature field by the numerical method, since today calculators have at their disposal the above commercial programs.

However, as a consequence of the fact that there exists the necessity of performing a parametric analysis of the constructed solution and the possibility of making accelerated calculations and that in most cases the solution of the temperature problem is a preparatory stage for estimating the thermal stresses, the control actions, and the optimization, it is required to obtain simple approximate analytical solutions without infinite series and solutions in explicit form.

In view of the above requirements, the inverse transform of (9) is constructed in the form of asymptotic expansions for small Fo (large s) and large Fo (small s).

Asymptotic Form of Solutions for Small Fo Numbers (large s). At the initial stage of heating of the coal particle, the solution is determined by the behavior of the transform (transfer function) $F_\mathrm{L}(R, s)$ in the range of large values of s. Let us represent $F_\mathrm{L}(R, s)$ in the form of the large-parameter expansion

$$F_\mathrm{L}\,(R, s) \cong \psi_1\,(R, s)\,\frac{\exp(-\sqrt{s})}{\sqrt{s}} + \psi_2\,(R, s)\,\frac{\exp(-2\sqrt{s})}{s} + \ldots\,. \tag{11}$$

Substituting (9) in view of (11) into the basic equation (5) and equating the terms at equal degrees of expansion, we obtain the following system of equations for finding ψ_1, ψ_2, ... :

$$R^{-2}\frac{\partial}{\partial R}\left(R^2\frac{\partial \psi_1}{\partial R}\right) - s\psi_1 = 0\,, \quad R^{-2}\frac{\partial}{\partial R}\left(R^2\frac{\partial \psi_2}{\partial R}\right) - s\psi_2 = 0, \ldots\,. \tag{12}$$

We find two boundary conditions for (12) from (7) and (8), respectively,

$$\psi_1'\big|_{R=0} = 0\,, \quad \psi_2'\big|_{R=0} = 0, \ldots ;$$

$$\psi_1'\big|_{R=1} = \sqrt{s}\,\exp\left(\sqrt{s}\right), \quad \psi_2'\big|_{R=1} = 0, \ldots .$$

Then the temperature field in the region of transforms with account for the first expansion term will be given in the form

$$\Theta_{\rm L}(R, s) - \frac{\Theta_0}{a} \cong 2{\rm Ki}_{\rm L}(s)\,\frac{\sinh\sqrt{s}\,R}{\sqrt{s}\,R}\,\exp\left(-\sqrt{s}\right) + \ldots . \tag{13}$$

Using the transform convolution theorem, we find the solution for the inverse transform with account for the first expansion term:

$$\Theta(R, {\rm Fo}) - \Theta_0 \cong \frac{1}{R}\int\limits_0^{\rm Fo}{\rm Ki}(\eta)\left\{\frac{1}{\sqrt{\pi\,({\rm Fo}-\eta)}}\exp\left[-\frac{(1-R)^2}{4\,({\rm Fo}-\eta)}\right]\right.$$

$$\left. - \frac{1}{\sqrt{\pi\,({\rm Fo}-\eta)}}\exp\left[-\frac{(1+R)^2}{4\,({\rm Fo}-\eta)}\right]\right\}d\eta\,. \tag{14}$$

For R values close to the surface layers, solution (14) is somewhat simplified:

$$\Theta(R, {\rm Fo}) - \Theta_0 \cong \frac{1}{R}\int\limits_0^{\rm Fo}{\rm Ki}(\eta)\,\frac{1}{\sqrt{\pi\,({\rm Fo}-\eta)}}\exp\left[-\frac{(1-R)^2}{4\,({\rm Fo}-\eta)}\right]d\eta\,. \tag{15}$$

Note that (13) has a singularity at the center of the sphere since $\sqrt{s}\,R \to 0$. In this case, the following expression holds:

$$\Theta(0, {\rm Fo}) - \Theta_0 \cong \int\limits_0^{\rm Fo}{\rm Ki}(\eta)\,\frac{1}{\sqrt{\pi\,({\rm Fo}-\eta)^3}}\exp\left[-\frac{1}{4\,({\rm Fo}-\eta)}\right]d\eta\,. \tag{16}$$

For the approximate solution (15), (16), let us represent Ki(η) by the following Taylor series:

$${\rm Ki}(\eta) \cong {\rm Ki}({\rm Fo}) + (\eta - {\rm Fo})\,{\rm Ki}'({\rm Fo}) + \ldots + \frac{\eta - {\rm Fo}}{n!}\,{\rm Ki}^n({\rm Fo})\,. \tag{17}$$

As a result, we obtain the solution for determining the dimensionless temperature with account for the first expansion term at small Fo:

$$\Theta(R, {\rm Fo}) - \Theta_0 \cong \frac{2}{\sqrt{\pi}\,R}\,{\rm Sk}\,[1 - \Theta^4(1, {\rm Fo})] + {\rm Bi}\,[1 - \Theta(1, {\rm Fo})]$$

$$\times\left[\sqrt{\rm Fo}\,\Phi_1(R, {\rm Fo}) + \frac{1-R}{2}\sqrt{\pi}\,\Phi_2(R, {\rm Fo})\right] + \ldots , \tag{18}$$

where

$$\Phi_1 = \exp\left[-\frac{(1-R)^2}{4{\rm Fo}}\right]; \quad \Phi_2 = {\rm erfc}\left(\frac{1-R}{2\sqrt{\rm Fo}}\right).$$

We find the surface temperature from the relation

$$[\Theta\,(1,\mathrm{Fo})-\Theta_0]\left[1-\Theta^4\,(1,\mathrm{Fo})+\frac{\mathrm{Bi}}{\mathrm{Sk}}\,[1-\Theta\,(1,\mathrm{Fo})]\right]^{-1}\cong 2\mathrm{Sk}\sqrt{\mathrm{Fo}}\left(\frac{1}{\sqrt{\pi}}+\frac{\sqrt{\mathrm{Fo}}}{3}+...\right). \tag{19}$$

Accordingly, near the symmetry center the solution will be written as follows:

$$\Theta\,(R,\mathrm{Fo})-\Theta_0\cong 2\left\{\mathrm{Sk}\,[1-\Theta^4\,(1,\mathrm{Fo})]+\mathrm{Bi}\,[1-\Theta\,(1,\mathrm{Fo})]\right\}\,\mathrm{erfc}\,\frac{1}{2\sqrt{\mathrm{Fo}}}\,. \tag{20}$$

Asymptotics of Solutions at Large Fo Numbers (small *s*). In this case, we represent the transfer function $F_{\mathrm{L}}(R,\ s)$ in (9) in the form of the small-parameter expansion of s:

$$F_{\mathrm{L}}\,(R,s)\cong\varphi_0\,(R,s)+s\varphi_1\,(R,s)+s^2\varphi_2\,(R,s)+...\,. \tag{21}$$

Substituting solution (9) in view of (21) into the basic equation (5) and equating the terms at equal degrees of s, we obtain the following linking system of equations for finding φ_0, φ_1, φ_2, ... :

$$R^{-2}\,\frac{\partial}{\partial R}\left(R^2\,\frac{\partial\varphi_0}{\partial R}\right)=0\,,\quad R^{-2}\,\frac{\partial}{\partial R}\left(R^2\,\frac{\partial\varphi_1}{\partial R}\right)=\varphi_0\,,\quad R^{-2}\,\frac{\partial}{\partial R}\left(R^2\,\frac{\partial\varphi_2}{\partial R}\right)=\varphi_1\,. \tag{22}$$

Each equation from (22) requires in the integration two boundary conditions for finding two constants.

Let us determine the conditions for finding the first integration constant of (22) from (8):

$$\varphi_0'\,|_{R=0}=0\,,\quad \varphi_1'\,|_{R=0}=0\,,\quad \varphi_2'\,|_{R=0}=0\,, \tag{23}$$

where the prime is the sign of the coordinate derivative.

The second integration constant is found from the integral relation, i.e.,

$$\int_0^1\frac{\partial}{\partial R}\left(R^2\,\frac{\partial\varphi_0}{\partial R}\right)dR=0\,,\quad \int_0^1\frac{\partial}{\partial R}\left(R^2\,\frac{\partial\varphi_1}{\partial R}\right)dR=\int_0^1 R^2\varphi_0 dR\,,\quad \int_0^1\frac{\partial}{\partial R}\left(R^2\,\frac{\partial\varphi_2}{\partial R}\right)dR=\int_0^1 R^2\varphi_1 dR\,. \tag{24}$$

Once φ_0, φ_1, φ_2, ... have been found, we can return from the region of transform (9) into the space of inverse transforms. As a result, the solution for the temperature function with account for the three terms will take the form

$$\Theta\,(R,\mathrm{Fo})-\Theta_0\cong 3\int_0^{\mathrm{Fo}}\mathrm{Ki}\,(\mathrm{Fo})\,d\mathrm{Fo}+\mathrm{Ki}\,(\mathrm{Fo})\,\frac{5R^2-3}{10}+\mathrm{Ki}'\,(\mathrm{Fo})\,\frac{1}{20}\left(\frac{R^4}{2}-R^2+\frac{27}{70}\right)+...\,. \tag{25}$$

The surface temperature (R = 1) is determined with the use of two terms from (25) from the Volterra equation of the second kind:

$$\Theta\,(1,\mathrm{Fo})-\Theta_0\cong 3\int_0^{\mathrm{Fo}}\left\{\mathrm{Sk}\,[1-\Theta^4\,(1,\mathrm{Fo})]+\mathrm{Bi}\,[1-\Theta\,(1,\mathrm{Fo})]\,d\mathrm{Fo}\right\}$$

$$+\frac{1}{5}\left\{\mathrm{Sk}\,[1-\Theta^4\,(1,\mathrm{Fo})]+\mathrm{Bi}\,[1-\Theta\,(1,\mathrm{Fo})]\right\}\,. \tag{26}$$

The solution of (26) is the expression

$$3\mathrm{SkFo}=f_0\left[\Theta\,(1,\mathrm{Fo}),\,\mathrm{Sk},\,\frac{\mathrm{Bi}}{\mathrm{Sk}}\right]-f_0\left[\Theta^*,\,\mathrm{Sk},\,\frac{\mathrm{Bi}}{\mathrm{Sk}}\right], \tag{27}$$

where

$$f_0(\mathrm{Sk}, p) = -\frac{\mathrm{Sk}}{5} \ln \left| 1 - \Theta^4 + p(1-\Theta) \right| + C_1 \left\{ C_1^{1/2} \ln \frac{C_2}{C_3} + \left(p + C_1^{3/2}\right) \left(|C_1 C_4| \right)^{-1/2} \ln \frac{C_5}{C_6} \right.$$

$$\left. - 2\left(p - C_1^{3/2}\right) \left(|C_1 C_7| \right)^{-1/2} \arctan \frac{2\Theta - C_1^{1/2}}{(C_1 C_7)^{1/2}} \right\} \left(p^2 + C_1^2\right)^{-1} ;$$

$$C_1 = \left\{ \frac{p^2}{2} + \left[\frac{p^4}{4} + \left(4\,\frac{1+p}{3} \right)^3 \right]^{1/2} \right\}^{1/3} + \left\{ \frac{p^2}{2} - \left[\frac{p^4}{4} + \left(4\,\frac{1+p}{3} \right)^3 \right]^{1/2} \right\}^{1/3} ;$$

$$C_2 = C_1^{1/2} \left[2\Theta\,(\Theta + C_1^{1/2}) + C_1 - p \right] \left(C_1^{3/2} + p \right)^{-1} ; \quad C_3 = C_1^{1/2} \left[2\Theta\,(\Theta - C_1^{1/2}) + C_1 + p \right]$$

$$\times \left(C_1^{3/2} + p \right)^{-1} ; \quad C_4 = 2p\left(C_1^{-1/2} - C_1 \right) ; \quad C_5 = 2\left(\Theta + C_1^{1/2} - C_4 \right)^{1/2} \left(C_1^{1/2} - C_4^{1/2} \right)^{-1} ;$$

$$C_6 = -2\left(\Theta + C_1^{1/2} + C_7 \right)^{1/2} \left(C_1^{1/2} + C_7^{1/2} \right)^{-1} ; \quad C_7 = -2p\left(C_1^{-1/2} + C_1 \right) ; \quad p = \frac{\mathrm{Bi}}{\mathrm{Sk}} .$$

The value of Θ^* is found from the relation $\Theta^* - \Theta_0 \cong \frac{1}{5}\mathrm{Sk}[(1 - \Theta^{*4}) + p(1 - \Theta^*)]$.

As a result of solving the fourth-order algebraic equation, we have

$$\Theta^* = -\frac{1}{2}(2C_8)^{1/2} + \left[(2 + 3\mathrm{Sk}^{-1} + p)\,(8C_8)^{-1/2} - \frac{1}{2} C_8 \right]^{1/2} , \tag{28}$$

where

$$C_8 = (A+B)^{1/3} + (A-B)^{1/3} ; \quad A = \frac{1}{4}\left(\frac{5}{\mathrm{Sk}} + p \right)^2 ; \quad B = \left\{ \left[\frac{1}{4}\left(\frac{5}{\mathrm{Sk}} + p \right) \right]^4 + \left[\frac{1}{3}\,\mathrm{Sk}^{-1} \left(\frac{5\Theta_0}{\mathrm{Sk}} + p \right) + \frac{1}{3} \right]^3 \right\}^{1/3} .$$

Now, knowing the surface temperature, we can determine the temperature in any cross section of the particle

$$\Theta(R, \mathrm{Fo}) \cong \Theta(1, \mathrm{Fo}) + \left\{ \mathrm{Sk}\,[1 - \Theta^4(1, \mathrm{Fo})] + \mathrm{Bi}\,[1 - \Theta(1, \mathrm{Fo})] \right\} \frac{(R^2-1)^2}{2} + \dots . \tag{29}$$

Below we give a number of partial solutions.

1. Regime of radiative heat supply to the particle:

small Fo

$$\Theta(R, \mathrm{Fo}) - \Theta_0 \cong \frac{2}{R}\,\frac{\mathrm{Sk}\,[1 - \Theta^4(1, \mathrm{Fo})]}{\sqrt{\pi}\,R} \left[\sqrt{\mathrm{Fo}}\,\Phi_1(R, \mathrm{Fo}) - \frac{1-R}{2}\sqrt{\pi}\,\Phi_2(R, \mathrm{Fo}) \right] , \tag{30}$$

here the surface temperature of the particle is calculated by the expression

$$[\Theta(1, \mathrm{Fo}) - \Theta_0]\left[1 - \Theta^4(1, \mathrm{Fo}) \right]^{-1} \cong 2\mathrm{Sk}\,\sqrt{\mathrm{Fo}} \left(\frac{1}{\pi} + \frac{\sqrt{\mathrm{Fo}}}{3} + \dots \right) ; \tag{31}$$

large Fo

$$\Theta(R, \mathrm{Fo}) - \Theta_0 \cong \Theta(1, \mathrm{Fo}) + \mathrm{Sk}\left[1 - \Theta^4(1, \mathrm{Fo}) \right] \frac{(R^2-1)}{2} , \tag{32}$$

here the surface temperature of the particle is determined by the formula

$$3\ \mathrm{Sk\ Fo} = f_1\,[\Theta\,(1, \mathrm{Fo})] - f_1\,(\Theta^*)\,, \tag{33}$$

where

$$f_1\,(\Theta) = -\frac{\mathrm{Sk}}{3}\ln\,(1-\Theta^4) + \frac{1}{4}\ln\frac{1+\Theta}{1-\Theta} + \frac{1}{2}\arctan\Theta\,.$$

2. The nonlinear boundary condition (7) is often linearized by introducing the effective heat-transfer coefficient by the additivity rule

$$\alpha_{\mathrm{ef}} = \alpha_{\mathrm{rad}} + \alpha_{\mathrm{con}}\,.$$

In this case, the calculation dependences will take the form:
for small Fo

$$\Theta\,(R, \mathrm{Fo}) - \Theta_0 \cong \frac{2}{R}\,\frac{\mathrm{Bi}_{\mathrm{ef}}\,[1-\Theta\,(1, \mathrm{Fo})]}{\sqrt{\pi}}\left[\sqrt{\mathrm{Fo}}\,\Phi_1\,(R, \mathrm{Fo}) + \frac{1-R}{2}\sqrt{\pi}\,\Phi_2\,(R, \mathrm{Fo})\right], \tag{34}$$

where

$$\Theta\,(1, \mathrm{Fo}) \cong \frac{\sqrt{\pi}\,\Theta_0 + 2\mathrm{Bi}_{\mathrm{ef}}\sqrt{\mathrm{Fo}}}{\sqrt{\pi} + 2\mathrm{Bi}_{\mathrm{ef}}\sqrt{\mathrm{Fo}}}\,;$$

for large Fo

$$\Theta\,(R, \mathrm{Fo}) - \Theta_0 \cong \frac{1}{2}\left(1 + \frac{2}{\mathrm{Bi}_{\mathrm{ef}}} - R^2\right)\left(\frac{1}{\mathrm{Bi}_{\mathrm{ef}}} + \frac{1}{5}\right)^{-1}\exp\,[-D\,(\mathrm{Bi}_{\mathrm{ef}})\,\mathrm{Fo}]\,, \tag{35}$$

where

$$D\,(\mathrm{Bi}_{\mathrm{ef}}) = 3\left(\frac{1}{\mathrm{Bi}_{\mathrm{ef}}} + \frac{1}{5}\right)^{-1};\quad \mathrm{Bi}_{\mathrm{ef}} = \frac{\alpha_{\mathrm{ef}} r_0}{\lambda}\,.$$

3. In the case of a "thermally thin body" (Bi < 0.5, Sk < 0.25), the calculation is carried out by the following dependences:

$$3\ \mathrm{Sk\ Fo} = f_2\,[\overline{\Theta}\,(1, \mathrm{Fo})] - f_2\,(\Theta_0)\,, \tag{36}$$

where

$$f_2\,(\overline{\Theta}) = A\ln\,(1-\Theta) - B\ln\,(\Theta - b_1) - \frac{1}{2}\,(A-B)\ln\,(\Theta^2 - \sqrt{2\alpha_0}\,\Theta + b_2)$$

$$-\frac{\sqrt{2}}{\sqrt{\alpha_1+\alpha_0}}\left[\frac{\sqrt{2\alpha_0}}{2}\,(A-B) + \frac{1}{b_1} - b_2\left(A - \frac{1}{b_1}\,B\right)\arctan\frac{\sqrt{2}\,\Theta - \sqrt{\alpha_0}}{\sqrt{\alpha_1+\alpha_0}}\right];$$

$$A = \left[(1-b_1)\left(1-\sqrt{2\alpha_0}+b_2\right)\right]^{-1};\quad B = b_1\left[(1-b_1)\left(b_1^2 - \sqrt{2\alpha_0} + b_2\right)\right]^{-1};$$

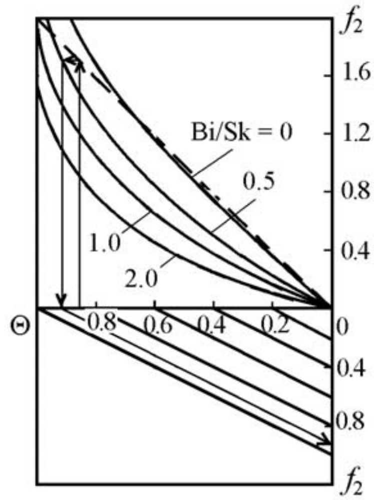

Fig. 2. Nomogram for determining the function f_2 depending on the parameters Θ and $\frac{\text{Bi}}{\text{Sk}}$ (the solution is shown by arrows).

$$\alpha_0 = \left[\frac{p^2}{16} + \left(\frac{p^4}{256} - \frac{q^3}{27}\right)^{1/2}\right]^{1/3} + \left[\frac{p^2}{16} - \left(\frac{p^4}{256} - \frac{q^3}{27}\right)^{1/2}\right]^{1/3} ;$$

$$p = \frac{\text{Bi}}{\text{Sk}} ; \quad q = -(1 + p) ; \quad \alpha_1 = \frac{p}{\sqrt{2\alpha_0}} ; \quad b_1 = \alpha_0 - \frac{\alpha_1}{2} ; \quad b_2 = \alpha_0 + \frac{\alpha_1}{2} .$$

For proximate calculations, the function f_2 is found from the nomogram of Fig. 2. Lines with arrows show an example of calculating f_2 depending on the parameters Θ = 0.828 and $\frac{\text{Bi}}{\text{Sk}}$ = 0.5; in the given case, f_2 = 0.833.

Nonstationary Thermoelastic Stresses in the Coal Particle. The solutions for the temperature field make it possible to obtain the calculation dependences for determining the thermal stresses, which is the basis for estimating the collapse of coal particles promoting the process of their burning.

The investigations carried out by Parcus [14] have shown that for a wide class of problems, with which the heating stage of the coal particle can be classified, temperature changes with time occur rather slowly, and this process can be considered as a certain sequence of equilibrium states. Such an approach is quasi-static and the time in a given problem plays the role of some parameter.

The coal particle is considered as a continuous sphere. Let us assume that this sphere deforms freely. In the case of the temperature field $\Theta(R, \text{Fo})$ having central symmetry, let us represent the stresses that arise in the form

$$\sigma_{rr} = \frac{2E\beta\,[\Theta\,(1, \text{Fo}) - \Theta\,(R, \text{Fo})]}{3\,(1 - \mu)} , \tag{37}$$

$$\sigma_{\varphi\varphi} = \frac{E\beta}{3\mu}\,[2\overline{\Theta}\,(1, \text{Fo}) + \overline{\Theta}\,(R, \text{Fo})] - 3\,\Theta\,(R, \text{Fo}) , \tag{38}$$

where

$$\overline{\Theta}\,(R, \text{Fo}) = \frac{3}{R^3}\int_0^R R^2\Theta\,(R, \text{Fo})\,dR ; \quad \overline{\Theta}\,(1, \text{Fo}) = 3\int_0^1 R^2\Theta\,(R, \text{Fo})\,dR .$$

TABLE 1. Results of the Dimensionless Time Calculations for Various Values of the Relative Surface Temperature at Small Fo

Dimensionless time	Method	$\Theta(1,\ \mathrm{Fo})$			
		0.212	0.2158	0.2230	0.2515
Fo	(31)	0.00028	0.0005	0.0011	0.0058
	[12]	0.0003	0.0005	0.001	0.005

TABLE 2. Results of the Calculation of the Relative Surface and Center Temperatures for Various Values of the Dimensionless Time at Large Fo

Relative temperatures	Method	Fo			
		0.3	0.7	1.3	2.5
$\Theta(0,\ \mathrm{Fo})$	(32)	0.474	0.769	0.933	0.994
	[12]	0.471	0.767	0.932	0.994
$\Theta(1,\ \mathrm{Fo})$	(32)	0.624	0.835	0.952	0.996
	[12]	0.634	0.839	0.953	0.996

As is known, any material resists tension less effectively than compression. Maximal tensile (positive) stresses under heating arise at the center of the particle ($R = 0$) and are found from the expressions

$$\sigma_{\varphi\varphi}^{\max} = \sigma_{rr}^{\max} = \frac{2E\beta}{3\,(1-\mu)}\,[\overline{\Theta}\,(1,\mathrm{Fo}) - \Theta\,(0,\mathrm{Fo})]\,. \tag{39}$$

As a result, we arrive at the following calculation relations for maximal stresses:

small Fo numbers

$$\begin{aligned}
\sigma_{\max} = {} & -\frac{2E\beta}{3\,(1-\mu)}\,[\mathrm{Sk}\,(1-\Theta^4\,(1,\mathrm{Fo})) + \mathrm{Bi}\,(1-\Theta\,(1,\mathrm{Fo}))]\,\frac{1}{\pi^{1/2}} \\
& \times \left\{\frac{1}{2} - 6\mathrm{Fo}\left[2\mathrm{Fo}^{1/2}\left(\exp\left(-\frac{1}{4\mathrm{Fo}-1}\right) + \mathrm{erf}\left(\frac{1}{2\mathrm{Fo}^{1/2}}\right)\right)\right]\right\} \\
& - \left\{8\mathrm{Fo}^{3/2} - \left(2\mathrm{Fo}^{1/2}\left[(8\mathrm{Fo}-1) + \pi^{1/2}\,\mathrm{erf}\left(\frac{1}{2\mathrm{Fo}^{1/2}}\right)\exp\left(\frac{1}{4\mathrm{Fo}}\right)(6\mathrm{Fo}-1)\right]\right.\right. \\
& \left.\left. \times \left(2\exp\frac{1}{4\mathrm{Fo}}\right)^{-1}\pi^{1/2}\right) - 2\,\mathrm{erfc}\left(\frac{1}{2\mathrm{Fo}^{1/2}}\right)\right\};
\end{aligned} \tag{40}$$

large Fo numbers

$$\sigma_{\max} = \frac{E\beta}{3\,(1-\mu)}\,[\mathrm{Sk}\,(1-\Theta^4\,(1,\mathrm{Fo})) + \mathrm{Bi}\,(1-\Theta\,(1,\mathrm{Fo}))]\,. \tag{41}$$

Calculation Analysis. To compare the approximate analytical formulas obtained in the present paper, let us make a comparison with the exact solution of the linear boundary-value problem at a boundary condition of the convective type Bi = const. The numerical values of the dimensionless parameters are as follows: $\mathrm{Bi} = 0.8$, $\Theta_0 = 0.2$. The Shivee-Ovoo coal is classified with brown coals of class B2. Unfortunately, the required body of experimental data on the thermal properties of this deposit has not yet been obtained. Therefore, the deficient parameters in the first approximation can be assumed to be identical to the parameters of the brown coals of the Kansk-Achinsk basin, also belonging to group B2, the available data on their properties being much more complete.

TABLE 3. Comparative Analysis of the Obtained Dependences for Thermal Stresses with the Exact Solution under Convective Heating of the Coal Particle

Thermal stresses	Computing method	Small Fo	Large Fo
σ_{rr}^{*}, $R = 0.5$	(34)	0.01965	0.1354
	[13, 14]	0.02122	0.1484
$\sigma_{\varphi\varphi}^{*}$, $R = 0.5$	(35)	0.01963	0.0903
	[13, 14]	0.2122	0.0961
$\sigma_{\max}^{*}$	(37)	0.4196	—
	(38)	—	0.1805
	[13, 14]	0.4452	0.2017

Note. $\sigma_{rr}^{*} = \sigma_{rr}\dfrac{3(1-\mu)}{2E\beta}$, $\sigma_{\varphi\varphi}^{*} = \sigma_{\varphi\varphi}\dfrac{3(1-\mu)}{2E\beta}$, $\sigma_{\max}^{*} = \sigma_{\max}\dfrac{3(1-\mu)}{2E\beta}$.

The computing experiment has been performed for a particle of size 10^{-2} m.

The following physical properties of the Shivee-Ovoo coal were taken: $\lambda = 0.254$ W/(m·K); $a = 1.63 \cdot 10^{-7}$ m^2/s; $\rho = 1.23 \cdot 10^{3}$ kg/m^3; $c = 1.22$ kJ/(kg·K); $\mu = 0.387$; $E = 5 \cdot 10^{9}$ N/m^2; $\beta = 10^{-5}$ K^{-1}.

The results of comparing the calculations are presented in Tables 1–3.

Conclusions. On the basis of the results of the comparisons made, we can draw the conclusion that the accuracy of the obtained approximate-analytical formulas is acceptable for engineering practice. The calculations have also shown that the temperature stresses for a Shivee-Ovoo coal particle of size 100 μm arising at a gaseous medium temperature of 1500°C do not exceed the ultimate tensile strength and particles of size up to 100 μm do not lose their continuity. But particles of size 10 mm and larger collapse at a medium temperature of 1000°C. In so doing, the collapse moment itself is achieved at the initial stage of heating (small Fo)

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NOTATION

a, heat diffusivity, m^2/s; Bi, Biot number; c, heat capacity, kJ/(kg·K); E, Young modulus, N/m^2; $F_{\mathrm{L}}(R, s)$, transfer function; Fo. Fourier number; Ki, Kirpichev number; $\mathrm{Ki}_{\mathrm{L}}(s)$, Laplace-transformed time function; s, transformation operator; R, dimensionless current radius; r, current radius of the particle, m; r_0, initial radius of the coal particle, m; Sk, Stark number; T, temperature, K; T_0, initial temperature of the particle, K; T_{s}, surface temperature, K; T_{m}, medium temperature, K; t, current time, s; t_{e}, initial time of drying, s; α, convective heat-transfer coefficient, W/(m^2·K); β, linear-expansion coefficient, 1/K; η, integration variable; λ, heat conductivity, W/(m·K); μ, Poisson coefficient; Θ_0, dimensionless initial temperature; $\Theta(R, \mathrm{Fo})$, dimensionless current temperature; $\Theta_{\mathrm{L}}(R, s)$, Laplace-transformed dimensionless temperature; σ, apparent coefficient of thermal radiation, W/(m^2·K^4); σ_{rr}, radial thermoelastic stresses, N/m^2; $\sigma_{\varphi\varphi}^{2}$, circumferential thermoelastic stresses, N/m^2; ρ, density, kg/m^3. Subscripts: 0, initial value of the parameter; con, convective; rad, radiant; e, evaporation; s, surface; w, medium; ef, efficient; L, Laplace transform symbol; rr, radial; $\varphi\varphi$, circumferential.

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